On the Origin of Planetary Nebula K648 in Globular Cluster M15

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ABSTRACT

We examine two scenarios for formation of the planetary nebula K648: a prompt scenario where the planetary nebula is ejected and formed immediately after a helium shell flash and a delayed scenario where a third dredge up occurs and the envelope is ejected during the following interpulse phase. We present models of both scenarios and find that each can produce K648-like systems. We suggest that the prompt scenario is more favorable but cannot rule out the delayed scenario.

Subject headings: globular clusters: individual (M15) — planetary nebulae: general — planetary nebulae: individual (K648) — stars: AGB and post-AGB

1. Introduction

The globular cluster M15 contains the well studied planetary nebula (PN) K648. This is one of the few Galactic PNe with a reasonably well-determined distance. Therefore, fundamental properties such as the stellar luminosity can be determined with some confidence. Because of its globular cluster membership, many of the progenitor properties, such as the zero age main sequence (ZAMS) mass, can be inferred reliably.

Due to the importance of K648 as a halo PN, it has been the focus of several abundance studies, and all of these show the abundances of most metals to be depleted relative to the sun, consistent with a progenitor of low metallicity. Carbon is an exception; studies which determine the ratio (by number) of C/O in K648 infer values that range from $4 - 11$ (Adams et al. 1984; Henry, Kwitter, and Howard 1996; Howard, Henry, and McCartney 1997), which is far above C/O in the Sun of 0.43 (Anders and Grevesse 1989, hereafter AG89). This is

in fact much higher than the average C/O ratio of ≈ 0.8 for solar neighborhood PNe (Rola and Stasińska 1994).

Low and intermediate mass stars that have left the main sequence, ascended the giant branch, and passed through the horizontal branch, then enter a thermally unstable phase where energy is generated by shell He and H-burning called the thermally pulsing asymtoptic giant branch (TP-AGB) stage, which is a very important yet not well understood phase [Detailed reviews of this stage can be found in Iben (1995), Lattanzio (1993), and Iben and Renzini (1983)]. During the TP-AGB stage the star alternates between a long stage where the luminosity is generated mostly by quiescent hydrogen shell burning, with a helium burning layer producing a minority of the energy, and a thermal runaway stage in the unstable helium burning layer (Schwarzschild and Härm 1965, 1967; and Weigert 1966). The second stage results in expansion of the outer layers and an extinguishing of the H burning shell. This short stage, characterized by rapid changes, with helium burning dominating the energy generation, is known as a thermal pulse or a He shell flash.

TP-AGB stars exhibit large mass-loss rates ranging from $10^{-7}-10^{-4}$ $M_\odot$ yr^{-1} . Indeed such high mass-loss rates are predicted to result in the ejection of the envelope, at which point the star leaves the AGB and becomes a planetary nebula central star (CSPN). The first models of CSPN tracks were made by Paczynski (1971) who showed that the CSPNs evolve horizontally on the HR diagram when nuclear burning is still taking place and then as they cool the luminosity and temperature decrease. Härm and Schwarzschild (1975) showed that a CSPN could leave the AGB as either a helium burning or hydrogen burning star. The observational consequences of hydrogen and helium burning have been studied in the more refined models including mass loss showed that the subsequent evolution of the central star depends on whether the star leaves the AGB as a helium or hydrogen burner [Schönberner (1981, 1983) and Iben (1984)].

Low mass stars ($M \lesssim 3$ $M_\odot$) can experience two mixing episodes or "dredge-ups". During dredge-up, material that has been processed by nuclear burning is mixed into the surface layers. At the entrance to the giant branch, the convective region can extend into the core, leading to mixing of CNO products into the outer layers. Similarly as shown by Iben (1975), after a thermal pulse on the AGB, the convective region can extend into the core, mixing He-burning products into the outer layers. These two mixing events are known as first and third dredge up, respectively (second dredge up will not concern us here). Therefore, a third dredge-up is a natural explanation of the high carbon abundance found in K648. On the other hand, no carbon stars have been observed either in M15 or in any other globular cluster, although such stars should be the immediate progenitors of objects such as K648 if a third dredge-up occurs. Thus, the lack of carbon stars in M15 weakens

the argument for a third dredge-up event.

One possible explanation for the absence of carbon stars is a delayed scenario in which the third dredge-up of carbon rich material changes the structure of the envelope during the following interpulse phase, ultimately increasing the mass-loss rate significantly and driving off the stellar envelope (Iben 1995). Thus envelope ejection is delayed until the interpulse phase following this dredge-up of carbon rich material.

Another explanation supposes that the envelope is removed during the quiescent He-burning stage that follows a thermal pulse (Renzini 1989 and Renzini and Fuci-Pecci 1988). The carbon then originates in a fast wind from the central star (CSPN). In addition, the wind produces shock-heating in the nebula, which, if not properly accounted for during an abundance analysis, may lead to the inference of a spuriously high C/O ratio. In this case the envelope would be ejected immediately after a thermal pulse while helium shell burning still dominates the luminosity. We refer to this mechanism as the prompt scenario.

In this paper we calculate detailed envelope models of thermally pulsing asymptotic giant branch star envelopes to test the predictions of the delayed mechanism, perform other calculations relevant to the prompt mechanism, and compare output of each with observations of K648. Section 2 describes the envelope code, section 3 presents the observational data and the results for the delayed and prompt models, and a brief discussion of our findings is given in section 4.

2. Models

The computer code used to calculate the delayed models is a significantly updated and modified version of a program kindly provided to us by A. Renzini for modeling the envelope of TP-AGB stars during the interpulse phase. Many of the basic details of the method are enumerated in Iben and Truran (1978) and Renzini and Voli (1981) and references therein; in this section we concentrate on those features which are different. In a future paper (Buell et al. 1997) we will provide a more detailed description of the code.

The mass of the hydrogen exhausted core (M_H) at the first thermal pulse is given by the expression found in Lattanzio (1986). During each interpulse phase the code follows the mass of the hydrogen exhausted core and envelope, the evolution of envelope abundances of ^4He, ^{12}C, ^{13}C, ^{14}N, and ^{16}O, and determines T_{eff} by integrating the equations of stellar structure from the surface to the core. Envelope abundances at the first pulse are determined by combining published main sequence levels with changes due to the first dredge-up. The former are established by scaling the AG89 solar abundances of all metals

except the alpha elements, i.e. oxygen, neon, and magnesium, to the appropriate metallicity, and then setting $[N_\alpha/Fe] = 0.4$, where N_α is the number abundance of O, Ne, and Mg. This last value is chosen from an examination of the trends in the data of Edvardsson et al. (1993) for [Fe/H]<-1.0 and by assuming that neon and oxygen vary in lockstep in PNe as shown by Henry (1989). The abundance changes due to the first dredge-up are calculated from the formulae of Groenewegen and deJong (1993).

The mass-loss both before and during the TP-AGB phase is very important, although the parameters are poorly understood. The pre-TP-AGB mass-loss is a free parameter, while during the TP-AGB phase, mass-loss is determined by using the expression of Vassiliadis and Wood (1993), which can be written as

$$\log \dot{M} = -11.43 + 1.0467 \times 10^{-4} \left(\frac{R}{R_\odot}\right)^{1.94} \left(\frac{M}{M_\odot}\right)^{-0.9} \ \ M_\odot \ \mathrm{yr}^{-1}. \tag{1}$$

The above rate is used until $\log \dot{M} = -4.5$, and then it is held fixed. Equation 1 is a $\dot{M}-$Period relation based on mass-loss from population I stars. However, recent calculations by Wilson, Bowen, and Struck (1995) suggest that the mass-loss rates of low metallicity AGB stars are also strongly dependent on radius. There is considerable uncertainty in this equation. For example, predicted mass-loss rates from other equations with a similar form (e.g. Bazan 1991) differ from predictions of eq. (1) by up to a factor of five.

The luminosity of TP-AGB stars after the first few pulses can be described by a linear relation between core-mass and luminosity as first discovered by Paczynski (1970). Models of TP-AGB stars have shown that for $M \lesssim 3.0 M_\odot$ this relation depends on metallicity (Lattanzio 1986, Hollowell and Iben 1988, Boothroyd and Sackmann 1988b). At the first pulse the luminosity of TP-AGB stars is less than the asymptotic core-mass-luminosity relation. The luminosity at the first pulse in our models is found by linearly extrapolating in metallicity from the expressions found in Boothroyd and Sackmann (1988b). After the first pulse, the luminosity of the AGB star rises steeply until it reaches a value predicted by the core-mass luminosity relation (CML) of Boothroyd and Sackmann (1988b). This relation predicts luminosity primarily as a function of core mass, although it has a weak dependence on helium and metal mass fractions.

Carbon rich material can be dredged from the core into the envelope following a thermal pulse. We assume that when the mass of the hydrogen-exhausted core exceeds a minimum mass ($M_{\mathrm{min}}^{\mathrm{DU}}$) that a dredge-up occurs. The amount of material dredged up, $\Delta M_{\mathrm{dredge}}$, is determined by the free parameter λ, where

$$\lambda = \frac{\Delta M_{\mathrm{dredge}}}{\Delta M_c}. \tag{2}$$

In eq. (2) ΔM_c is the amount of core advance during the preceding interpulse phase. We determine the composition of the dredged up material from the formulas in Renzini and Voli (1981), with $^4He \approx 0.75$, $^{12}C \approx 0.23$, and $^{16}O \approx 0.01$ as the approximate mass fractions.

Finally, the code uses the opacities of Rogers and Iglesias (1992) supplemented by the low temperature opacities of Alexander and Ferguson (1994).

3. Results and Discussion

3.1. Observational Parameters

Numerous observed and inferred parameters for K648 are listed in Tables 1a and 1b, where the symbols in column (1) are explained in the table notes. We comment here on the method of determination for several of them.

Radio images of K648 have been made by Gathier et al. (1983) and optical images were made by Adams et al. (1984), and recently by Bianchi et al. (1995) using the HST. The HST data called into question the small size for the nebula inferred in the radio studies of Gathier et al. (1983) and the optical studies of Adams et al. (1984), since HST was able to resolve the structure of the nebula. This leads to, e.g., a larger planetary mass and smaller electron density. In Tables 1a and 1b, we quote all results.

M_{PN} was computed using equation V-7 in Pottasch (1984), while the dynamical age was estimated by dividing the nebular radius by the expansion velocity (v_{exp}). Since no v_{exp} is available for K648 we use a range which represents typical values for PNe. The central star mass for K648 was estimated by linearly interpolating/extrapolating using both hydrogen burning and helium burning post-AGB tracks of Vassiliadis and Wood (1994) in the log L-log T plane. The metallicity of M15 suggests using a low Z track, although the carbon abundance of K648, if correct, would increase the metallicity of the star, suggesting that a higher Z track is more appropriate. Since the metallicity dependence is unclear, we estimated the range of possible central star masses by performing the interpolation for each metallicity considered by Vassiliadis and Wood. Thus, the mass range of the central star is 0.55–$0.58 M_\odot$ for the H burning tracks and 0.56–$0.61 M_\odot$ for the He-burning tracks. We adopt a final core mass of $0.58 \pm 0.03 M_\odot$.

The theoretical age of the central star was estimated from the figures of Vassiliadis and Wood and linearly interpolating in $\log L$ between tracks which closely match the core mass of K648, e.g., the hydrogen burning M_c=0.56, Z=0.016 track and the helium burning

M_c=0.56 $M_\odot$, Z=0.004 give evolutionary ages of $\sim$ 12000 yr and $\sim$1800 yr, respectively. Other tracks with $M_c \lesssim 0.6 M_\odot$ and different metallicities give similar results. When compared to the dynamical age a He burning track is favored.

The adopted abundances of K648 for He/H, C/O, and N/O ratios represent a range of recent literature values. Howard et al. (1997) find that in six of the nine halo PNe they studied, the C/O ratio exceeds the solar value. Many of these nebulae have stellar temperatures much higher than that of K648, implying that they are older and more evolved. Since the high C/O ratios persist into the later stages of PN evolution, this suggests that the inferred C/O is not influenced by the presence of shock heating in the nebula.

The mass-loss rate at the tip of the AGB was determined by dividing the nebular mass by the dynamical age. This is in reality a lower limit since it assumes that the nebula has a filling factor of 1, which is unrealistic. By this procedure we calculate that the lower limit to the mass-loss is 9×10^{-6} $M_\odot$ yr^{-1}. The upper limit is assumed to be 10^{-4} $M_\odot$ yr^{-1}.

The composition of the central star is uncertain, as two recent papers do not agree. McCarthey et al.(1996) find that the central star has a normal helium abundance, whereas Heber et al.(1993) find that the central star is helium and carbon rich.

3.2. Delayed Scenario

We have calculated several low mass, low metallicity models, but here we focus on the two models listed in Table 2, where we present the model input parameters: the ZAMS mass (M), the core mass at PN ejection (M_c), the mass of the PN (M_{PN}), the ZAMS [Fe/H] ratio, the adopted ratio of the mixing length to pressure scale height (α), the mass of the model star at the first thermal pulse (M_{FTP}), the adopted dredge-up parameter (λ), and the minimum core mass for dredge-up ($M_{c,min}^{DU}$). The panels of Figure 1 show the evolution of the interpulse luminosity, the stellar radius, the mass-loss rate, and the core mass as a function of total mass. All quantities are expressed in solar units. Figure 2 shows the evolution of the chemical composition of the envelope as a function of total mass. Table 1b compares the observed quantities to our predicted ones.

We note in Figure 1 that the interpulse radius of each model star increases dramatically after the final pulse, as compared to the preceding interpulse phase. The increase in radius leads to a large increase in the mass-loss rate in each model during the final interpulse phase; the mass-loss rate increases by almost a factor of 100 in model 2 and by a factor of 5 in model 1. This is a consequence of the steep dependence of our mass-loss law on

the stellar radius. The significant increase in the mass-loss rate causes the star to lose its envelope in a few thousand years. The mass-loss rate for model 1 is clearly too small relative to the observationally derived value. However, we have found that by reducing the mixing length (α) by a factor of two, as we have done in model 2, we can make a model that essentially reproduces the observed AGB tip mass loss rate.

The significant event that occurs during the final pulse is a dredge-up of helium and carbon rich material. The mass of material dredged up is a few times $10^{-5} M_\odot$. However, given the mass of the envelope and the low initial abundances, the amount of carbon dredged into the envelope is significant enough to increase the carbon mass fraction by a large factor in each case. Consequently, the envelope opacity rises, causing a dramatic increase in the stellar radius.

The envelope of each model at the last thermal pulse is only a few times $10^{-2} M_\odot$ and, after the final carbon dredging pulse, is ejected on a timescale of a few hundred years. Each model star is a carbon star for only a few hundred years, due to the rapid mass-loss after a dredge-up of carbon. This short lifetime, coupled with the relatively low incidence of PNe in globular clusters [two confirmed and three possible candidates (Jacoby et al. 1995)], perhaps explains why carbon stars have not been observed in globular clusters.

An important check on our models is to compare the predicted AGB tip luminosity with its observed value. The predicted luminosity of our models at the top of the AGB agrees fairly well with the tip of M15's red giant branch (Adams et al. 1984). Our models suggest that the observed AGB tip will actually correspond to the second-to-last pulse, since after the dredge-up event the star is predicted to remain as an AGB star for only $\sim$1000 yr. The luminosity of K648 in Adams et al. (1984) appears to be 0.1dex higher than the tip of the giant branch, this may be due to the metallicity enhancement due to the dredge up. As noted earlier, the core-mass luminosity relationship depends on metallicity, with the luminosity at a set core-mass increasing with increasing metallicity. If we lower the luminosity still further to $\sim$2000L$_\odot$ to match the tip of the giant branch, we believe that the addition of carbon to the envelope will still cause envelope ejection.

There is some question about whether or not dredge-up can occur at the low values of $M_{c,min}^{DU}$ indicated by our models (see Table 2). While Lattanzio (1989) found that dredge-up can occur at a core mass above $0.605 M_\odot$, the same study also found a dependence of the minimum dredge-up mass on metallicity, with lower metallicities giving lower mass dredge-ups. Boothroyd and Sackmann (1988c) found that if they increased the mixing length parameter α from 1 to 3, they were able to cause a dredge-up in a model with Z=0.001, $M_c = 0.566 M_\odot$, and M = 0.81M$_\odot$, although it is unclear if a mixing length this large is justified. Additionally, to match the low luminosity end of the carbon star luminosity

function of the LMC, Groenewegen and deJong (1993) had to set $M_{min}^{DU} = 0.58 M_{\odot}$. From these studies it appears that our values for $M_{c,min}^{DU}$ are not unreasonable.

Finally, we point out that each of the delayed models gives a very natural explanation of the high carbon abundance of K648 and the lack of carbon stars. Each model also predicts the observed mass of the ionized gas to be a few times $10^{-2} M_{\odot}$. The C/O ratios of each model range from 4 to 25, with model 2 giving the best fit, which agrees reasonably well with the observed values of $4 - 11$. The He/H ratios of the model stars also agree with the observed value of 0.09. The high N/O ratio inferred in the models may be an artifact of our choice of initial O abundance and hence could be reduced with a higher O abundance, which would also slightly reduce the C/O ratio. Thus, our delayed models are consistent with several important observed properties of the K648 system.

3.3.   Prompt Scenario

An alternative scenario results if we apply our mass-loss formulation to the secondary luminosity peak (SLP) which follows the helium shell flash of the $1 M_{\odot}$, Z=0.001 model of Boothroyd and Sackmann (1988a, BS88a). The metallicity of the BS88a model is a factor of $\sim$5 higher than M15, however, no models of the appropriate metallicity exist and we attempted to use the closest one in terms of Z, M_c, and M. The SLP corresponds to the region between point C and the vertical dashed line on figure 2 of Boothroyd and Sackmann (1988a), i.e. the same place that Renzini (1989) and Renzini and Fuci-Pecci (1988) predict this event to occur when the star expands. The SLP occurs when the excess luminosity produced in the helium shell flash reaches the surface. This peak can be seen in most models of low mass AGB stars [Iben(1982), BS88a, VW93].

It should be noted that this is the point where dredge-up can occur, although it does not necessarily do so. This scenario does not require a dredge-up of carbon rich material for envelope ejection. We define the prompt scenario as ejection at the SLP without the dredge-up of carbon rich material.

The adopted parameters of this model are shown in Table 3. The luminosity and radius are eyeballed lower limits from the SLP of BS88a, while the mass and core mass are parameters stated in their text. The mass-loss rate calculated from our prescription [i.e. eq. (1)] is $\sim 10^{-5} M_{\odot}$ yr^{-1} (essentially the Eddington limit), which will remove the $0.03 M_{\odot}$ envelope in a few thousand years. This model is similar to K648 in terms of core mass and envelope mass. Values for luminosity, radius, mass-loss rate, and core mass for the prompt scenario are indicated with filled diamonds in Fig. 1 and observed quantities are

also compared to those predicted for this scenario in Table 1b.

A carbon-rich nebula could be formed by the prompt mechanism if a sufficient amount of helium and carbon-rich material is ejected during the post-AGB phase and mixed with the ejected hydrogen-rich envelope. The fast wind overtaking the slower wind will produce a shock which would likely be Rayleigh-Taylor unstable, causing the nebula to mix. Only $5 - 15 \times 10^{-5}$ M$_\odot$ of material with mass fractions of ^4He=0.75 and ^{12}C=0.23 needs to be mixed into the envelope to match the C/O ratio of K648. Carbon-rich material can be ejected into the nebula during the post AGB phase. As a star moves horizontally across the HR diagram from the AGB stage to CSPN position, the mass-loss rate will decrease as the wind speed increases, so the material ejected during this transition can be mixed with the slower hydrogen rich envelope. And since the currently observed mass-loss rate of the K648 central star is $10^{-9} - 10^{-10}$ M$_\odot$ yr^{-1} (Adams et al. 1984; Bianchi et al. 1995), the nebula is no longer being polluted. Examination of the models of Vassiliadis and Wood (1994) suggests that as the star moves from the AGB phase to the CSPN phase, the mass loss rate drops from 10^{-5} M$_\odot$ yr^{-1} to $\sim 10^{-10}$ M$_\odot$ yr^{-1}, indicating that during this transition the mass loss rate was higher in the past, and possibly high enough to account for the carbon enrichments in K648.

In the prompt scenario, the envelope is ejected when the star is burning helium and as a result the resulting CSPN will be follow a helium burning track (Schönberner 1981, 1983; Iben 1984).

Thus, in the prompt scenario, the evolved star ejects sufficient carbon into a slower moving hydrogen-rich shell to produce the PN we observe today. Mixing is assumed to occur due to shock induced instabilities. Since the prompt scenario postulates the removal of the entire H-rich envelope during the He burning stage, we expect K648 to follow a He burning track because the H-burning shell has been extinguished during the thermal pulse. Ultimately, a white dwarf of type DB will be produced.

4.   Discussion

One additional scenario is again a delayed one, but one in which the CSPN is a helium burner. We have not as yet performed calculations relevant to it. In this case, if a dredge-up occurs, it does so at the SLP. The stellar envelope will be enriched in carbon and the added opacity may allow an even greater expansion during the SLP, making it more likely that the envelope will be ejected during this phase. The resulting PN would be carbon rich and have a helium burning CSPN. We feel that this is also a promising model, although, proper

calculations of this scenario need to be done.

Both the prompt and delayed scenarios can be made to match many of the observed features of K648. With each mechanism the radius increases dramatically: in the prompt because of the increase in luminosity of the star after a thermal pulse and in the delayed because of an increase in the opacity due to an infusion of carbon rich material. In addition, both mechanisms produce ^{12}C in sufficient amounts to explain the observed C/O ratio.

The most serious difficulty with the prompt scenario is that it can only explain the enhancement of the carbon and helium abundances by essentially adhoc means, in this case assuming the central star wind pollutes the rest of the nebula or by shocks and carbon-rich pockets due to this wind. This may not be an unreasonable assumption, since the mass of K648 is low compared to a "typical" PN ($\sim 0.1 M_\odot$). To test the prompt scenario would require a detailed model following the star from the horizontal branch to the central star phase with attention to the details of mass-loss to see if the central star wind can truly enhance the carbon and helium abundances of the PN and multidimensional hydrodynamics to test the mixing hypothesis.

The difficulty with the delayed scenario is it predicts that the CSPN should be a H-burner. The dynamical age of K648 favors a He-burning CSPN which is more likely to occur in the prompt scenario as the envelope is ejected during a phase when helium burning is dominant. Since we assume that for a given metallicity only one of these scenarios will be operative, a strong observational test to determine the correct scenario would be to search for white dwarfs in M15. If they are found to be type DB, this would favor the prompt scenario, and if they are type DA, the delayed scenario is more likely.

A point favoring the prompt scenario is that it naturally accounts for the dynamical age. On the other hand, this scenario requires the assumption of efficient mixing and there is some evidence (cf. section 3.1) that signatures of the requisite shocks are not actually observed. However, until detailed models are produced, both remain as viable evolutionary scenarios for K648 and similar systems.

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Fig. 1.— Shown in the panels of this figure are the evolution of our thermally pulsing AGB models and the parameters of our prompt model. The dotted line and open squares track model 1, the solid line and open circles track model 2, and the solid diamonds are the parameters of the prompt model. The abscissa of the graphs tracks the mass of the models in solar masses. Due to mass loss the stars move from right to left on the graphs. The parameters in the panels are for the interpulse phase. From top to bottom the parameters are stellar luminosity (in $L_\odot$), radius (in $R_\odot$), the mass-loss rate (in $M_\odot$ yr^{-1}), and the mass of the core. The observed upper and lower limits of the AGB tip luminosity are indicated with dark long dashed lines, the lower limit on the AGB tip mass-loss rate is indicated with a long dashed line, and the upper and lower limits of the central star mass are indicated with long dashed lines.

Fig. 2.— Shown in the panels of this figure is the evolution of the surface abundance ratios. The symbols have the same meaning as the first figure. In the C/O panel the upper and lower observational limits are shown on the figure with the dark long dashed line. The range of possible He/H and N/O is encompassed by the ordinates of these figures.

Table 1a.   Observational Data for K648

Parameter	Value	ref.
T_{eff}	36000 ± 4000 K	1,2
d	10.0 ± 0.8 kpc	3
θ	$1.0 - 2.5''$	1,2,5
v_{exp}	$15 - 25$ km s^{-1}	
n_e	$1700 - 8000$ cm^{-3}	1,2
T_e	12000 K	1,2
$\log F_{H\beta}$	-12.10 ± 0.03	4

References. — (1) Adams et al. 1984; (2) Bianchi et al. 1995; (3) Durell and Harris 1993; (4) Acker et al. 1992; (5) Gathier et al. 1983

Note. — This table is a summary of the observed and inferred parameters for PN K648. The effective temperature, T_{eff}, refers to the central star, while the distance, d, is the adopted distance to K648. The following nebular parameters are also listed: the angular size of the nebula, θ; the expansion velocity, v_{exp}; the electron density, n_e; the ionized gas temperature, T_e; and the log of the measured Hβ flux in erg cm^{-2} s^{-1}. The large range in θ and n_e arise from differences between newer HST data and ground based data, The HST data give higher a value of θ and a lower value for n_e.

Table 1b.   Observational Data and Models Compared

Parameter	Observed Value	ref.	Delayed Scenario	Prompt Scenario
L	$3200 - 4700\ L_{\odot}$	1,2	4600	4000
M_{PN}	$0.015 - 0.090\ M_{\odot}$	1,2	$0.048 \pm 0.012\ M_{\odot}$	$0.064\ M_{\odot}$
M_c	0.58 ± 0.03		0.57 ± 0.01	0.58
τ_{dyn}	$2000 - 8000$ yr		12000 yr	1800 yr
He/H	$0.083 - 0.10$	1,3,4	$0.087 - 0.091$	0.9
C/O	$4 - 11$	1,3,4	$4 - 25$	4
N/O	$0.05 - 0.20$	1,3,4	$0.17 - 0.19$	0.17

References. — (1) Adams et al. 1984; (2) Bianchi et al. 1995; (3) Henry, Kwitter, and Howard 1996; (4) Howard, Henry, and McCartney 1997

Note. — This table compares the observed and predicted parameters for PN K648. The observed luminosity, L, refers to the central star, while the predicted luminosity is the luminosity on the AGB, but since the tracks are nearly horizontal they should be comparable. The following nebular parameters are also listed: the mass of ionized gas in the nebula, M_{PN}; the mass of the central star, M_c; the dynamic timescale, τ_{dyn}; and the abundance ratios He/H, C/O, and N/O by number. The abundances for the prompt scenario are calculated assuming $0.00014 M_{\odot}$ of helium and carbon rich material is removed by mass-loss from the CSPN. The observed value for the dynamical timescale, τ_{dyn}, corresponds to an upper limit for the age of the nebula. The theoretical values correspond to evolutionary time scales required to reach a given central star temperature. The large range in L and M_{PN} arise from differences between HST data and ground based radio and optical data. The HST data give higher values for L and M_{PN}.

Table 2.   Input Parameters and Results for Delayed Models

No.	M	M_c	M_{PN}	[Fe/H]	α	M_{FTP}	λ	$M_{c,min}^{DU}$
1	0.88	0.56	0.037	-2.1	1.6	0.62	0.10	0.55
2	0.85	0.58	0.060	-2.1	0.8	0.72	0.02	0.56

Note. — Masses are in $M_\odot$

Table 3.   Adopted Prompt PN ejection parameters

Parameter	Value
Luminosity	4000 $L_\odot$
Radius	400 $R_\odot$
Mass	0.58 $M_\odot$
Core Mass	0.54 $M_\odot$
Mass-Loss Rate	$3.2 \times 10^{-5}\,M_\odot\ \mathrm{yr}^{-1}$
Time in Stage	2000 yr

Note. — The values in this table are estimated from Figure 2 of BS88a for a 1.0 $M_\odot$, Z=0.001 model. All values are appropriate between point C and the vertical dashed line on this figure. The radius and luminosity are the estimated lower limits. The mass and core mass are taken from their listed values. The mass-loss rate is calculated from our mass-loss prescription. The time in this stage is estimated from the BS88a graph.